**Number theory**

This booklet will introduce some of the important ideas in number theory and some formal notation.

**Natural numbers** $N$

The set of natural numbers, also called the counting numbers, $N$= {1, 2, 3, 4, ...}

It is common, although not necessary to use the letter n when referring to an element of the natural set.

**Task**

What numbers belong to the set 2n? Where $n\in N$

What numbers belong to the set 3n? Where $n\in N$

What numbers belong to the set 5n? Where $n\in N$

What expression represents the set of multiples of ten {10, 20, 30, …}?

What expression represents the set of odd numbers {1, 2, 3, …}?

Prove that the sum of two even numbers is even.

Prove that the sum of two odd numbers is even.

Prove that the product of two even numbers is even.

Prove that the product of two odd numbers is odd.

Consider two numbers in the natural set, using addition and subtraction only give examples of some calculations that can be done. Is the result always in the natural set?

**Integer numbers** $Z$

The integer numbers are the set {... -4, -3, -2, -1, 0, 1, 2, 3, 4, …}

**Task**

Consider two numbers in the integer set, using addition, subtraction, multiplication and division only give examples of some calculations that can be done. Is the result always in the integer set?

**Rational numbers** $Q$

Rational numbers are of the form $\frac{a}{b}$ where a, b belong to $a, b \in Z$

**Task**

Consider two numbers in the rational set, using addition, subtraction, multiplication, division and indices only give examples of some calculations that can be done. Is the result always in the rational set?

**Real numbers** $R$

**Task**

Consider two numbers in the real set, using addition, subtraction, multiplication, division and indices only give examples of some calculations that can be done. Is the result always in the real set?

**Task**

Construct a Venn diagram to show the sets $N, Z, Q, R$

**Divisibility rules**

**Task**

Prove each of the following divisibility rules.

Divisibility by 2: A natural number is divisible by two if the last (units) digit is divisible by two.

Divisibility by 3: A natural number is divisible by three if the sum of the digits is divisible by three.

Divisibility by 4: A natural number is divisible by four if the last two digit number is divisible by four.

Divisibility by 5: A natural number is divisible by five if the last (units) digit is divisible by five.

Divisibility by 6: A natural number is divisible by six if it is divisible by two and three.

Divisibility by 8: A natural number is divisible by eight if the last three digit number is divisible by eight.

Divisibility by 9: A natural number is divisible by nine if the sum of the digits is divisible by nine.

**Prime numbers**

Definition: A prime number is a natural number with exactly two distinct factors, the first few primes are 2, 3, 5, 7, 11, 13, 17, 19, 23, …

**Task**

Is 91 prime? How do you know?

Is 71 prime? How do you know?

Write an algorithm (set of instructions) for determining whether a number is prime, try to be as efficient as possible.

On the 100 square below highlight all the prime numbers, describe your method for finding the primes efficiently.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

On the grid below highlight all the primes. What do you notice?

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 1 | 2 | 3 | 4 | 5 | 6 |
| 7 | 8 | 9 | 10 | 11 | 12 |
| 13 | 14 | 15 | 16 | 17 | 18 |
| 19 | 20 | 21 | 22 | 23 | 24 |
| 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 |
| 37 | 38 | 39 | 40 | 41 | 42 |
| 43 | 44 | 45 | 46 | 47 | 48 |
| 49 | 50 | 51 | 52 | 53 | 54 |
| 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 |
| 67 | 68 | 69 | 70 | 71 | 72 |
| 73 | 74 | 75 | 76 | 77 | 78 |
| 79 | 80 | 81 | 82 | 83 | 84 |
| 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 |
| 97 | 98 | 99 | 100 | 101 | 102 |
| 103 | 104 | 105 | 106 | 107 | 108 |
| 109 | 110 | 111 | 112 | 113 | 114 |
| 115 | 116 | 117 | 118 | 119 | 120 |

How many prime numbers are there? Can you prove this?

**Prime factorisation**

The fundamental theorem of arithmetic states that any natural greater than 1 is either prime or can be expressed as a product of primes uniquely. For example;

$$4125=3×5^{3}×11$$

**Task**

Consider the prime factorisation of square numbers, cube numbers, powers of four, ect. Make conjectures and prove.

**Task**

Consider the prime factorisation of these numbers and the number of factors each has. Can you find a relationship?

1. 5
2. 10
3. 20
4. 40
5. 80
6. 400
7. 2000
8. 22000